- Legendrian Knot $\rightsquigarrow$
  - DGA up to stable tame isomorphism
  - LCH

Q: What are some properties of this invariant?

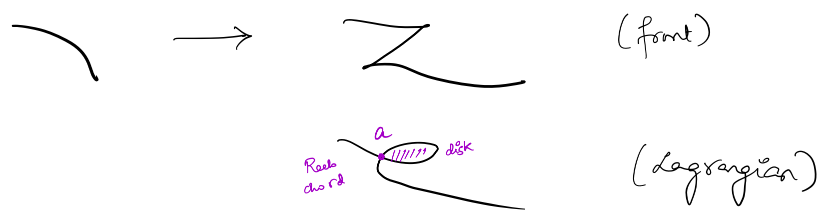
- relationship b/w DGA upto stable tame isomorphism and LCH
- value known for certain knots
- is it a complete invariant? if not what are its limitations?
- can it really give a lot of useful information?

- **Prop 1:** If Λ is a stabilised Legendrian knot then the LCH of Λ is trivial.

Reason: "Stokes' thm" $\Rightarrow$ signed sum of lengths of Reeb chords = area of the disk.

$$| d \in \Delta(a; b_1, \dots, b_n) | = h(a) - \sum_{i=1}^{n} h(b_i)$$

Stabilising :



Can ensure $h(a) < h(b)$ for all chords b.

Then $\partial a = 1$. $|a| = 1$

$\Rightarrow$ for any h s.t. $\partial h = 0$,

$$\partial(ah) = h\partial a + (-1)^{|a|} a \partial h = h$$

$\Rightarrow$ h is in kernel $\Leftrightarrow$ h in boundary.

So LCH vanishes.

■

- Algebraic question about DGA :

If two DGAs have isomorphic homology, are they stable tame isomorphic?

Answer : NO

- Firstly, we see that any two stabilised knots have isomorphic LCH.

Now we show their DGAs can be distinct, even up to stable tame isomorphism.

Fact : # even-graded generators − # odd-graded generators = Thurston-Bennequin number.

$\Rightarrow$ tb should be same for knots whose DGAs are stable tame iso.

But there are stabilised knots with different tb's.

∎

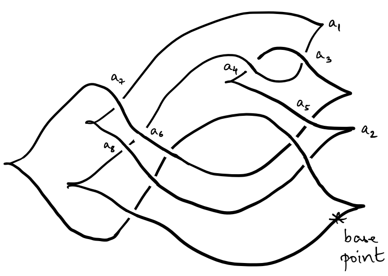
But that is the only difference. If two Chekanov-Eliashberg DGAs have vanishing homology and the knots have same tb, the DGAs are stable tame isomorphic.

Proof: Rmk 3.13 of Etnyre-Ng notes.

• Q: A natural converse to Prop 1: If $LCH(\Lambda)$ vanishes, is Λ stabilised?

A: No (Sivek '13)

Ex:



This knot has vanishing LCH.

$\partial(a_1) = 1 + a_8 + a_8 a_4 a_3$

$\partial(a_2) = 1 + a_5 a_7$

$\partial(a_6) = -a_7 a_8$

$\partial(a_3) = \partial(a_4) = \partial(a_5) = \partial(a_6)$

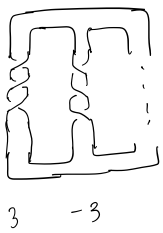
$= \partial(a_7) = 0.$

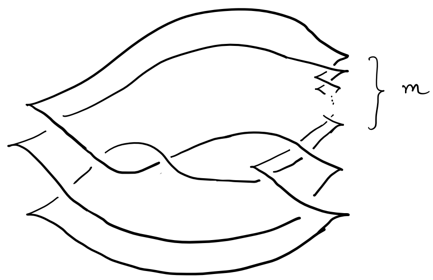
Then show $\partial(a_2 a_8 + a_5 a_6) = a_8$. (Use Leibniz rule)

$\Rightarrow \partial\left(a_1 - (a_2 a_8 + a_5 a_6)(1 + a_4 a_3)\right) = 1.$

• Fact: The Chekanov-Eliashberg DGA does not characterise the unknot.
(Limitation)

$\forall\, m \geq 1$, a Leg. representative of the pretzel knot $P(3,-3,-3-m)$ has a DGA which is stable tame isomorphic to the DGA of standard Legendrian knot.



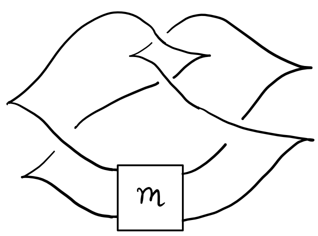


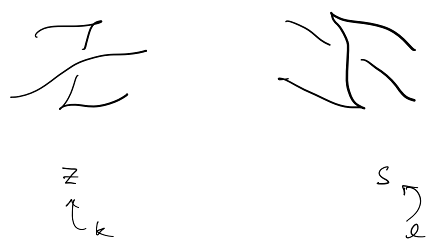
($m=1$: Appendix A, Etnyre–Ng)

(Limitation)

- Given any Legendrian knot Λ, one can produce arbitrarily many Legendrians whose DGA is stable tame iso. to the DGA for Λ, by taking connected sum of Λ with the above family.

(Success)





Legendrian twist knot.

Fix m: $tb = 1$

$rot = 0$.

Linearised contact homology distinguishes distinct pairs (unordered) $\{k, \ell\}$, where $k = \# Z$

$\ell = \# S$