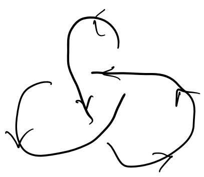
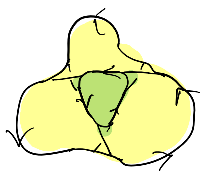
- Surfaces bound by knots :

1) Seifert's algorithm

2) Ribbon knots and slice surfaces

- 1) e.g.





2 disks + 3 bands

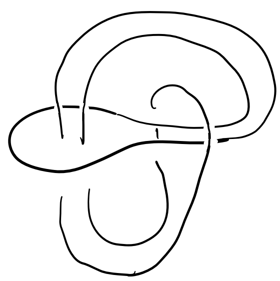
↪ once-punctured torus (by classification of surfaces)

surface in S^3

$\chi = \#\text{ disks} - \#\text{ bands}$

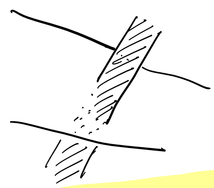
$= 2 - 2g - 1$ (g : genus)

2) e.g.



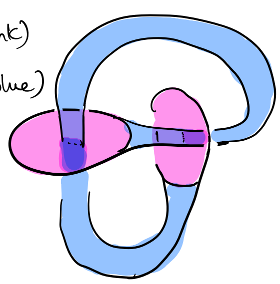
Stevedore knot (6_1).

Ribbon knot



these self-intersections in surfaces can be pushed apart in 4 dimensions

2 disks (pink) + 1 band (blue) = disk



surface in B^4 without self-intersection

→ if knot can be described by n disks and $n-1$ bands, it is slice (also ribbon)

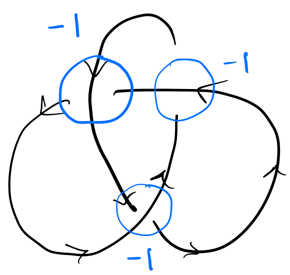
$g(K)$:= min genus of Seifert surface

$g_4(K)$:= min genus of slice surface

- <u>Linking number and writhe</u>.

<u>Writhe</u>: For an oriented knot diagram, signed sum of crossings. (not a knot invariant)

e.g.

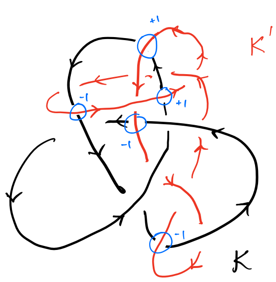


writhe = −3.

<u>Linking number</u>: Given knots K and K',

$lk(K, K')$ = signed count of all crossings where K goes over K' (or vice-versa)

e.g.



crossings where K' goes over K: 5

$lk(K, K') = +2-3 = -1$

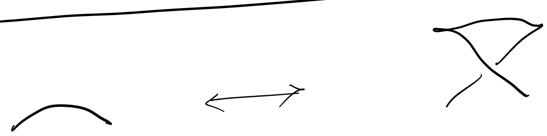
- **Legendrian knots** : image of $\varphi : S^1 \to \mathbb{R}^3$

  satisfies $z'(\theta) - y(\theta)\, x'(\theta) = 0$.

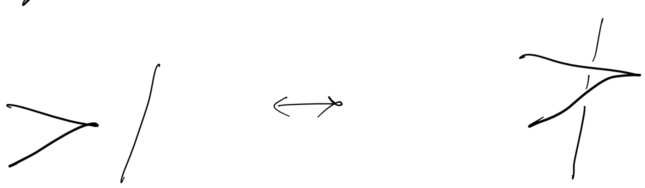
- Front projections : $\Pi : (x, y, z) \to (x, z)$
  - no vertical tangencies. no crossings.
  - given a knot diagram in $\mathbb{R}^2$ without vertical tangencies, can choose parametrisation to get a well-defined Legendrian corresponding to it.

- Legendrian Reidemeister moves :

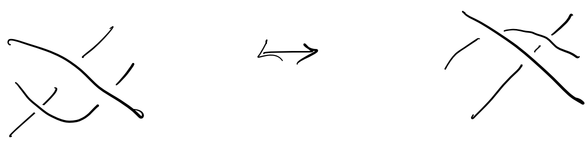
R1



R2

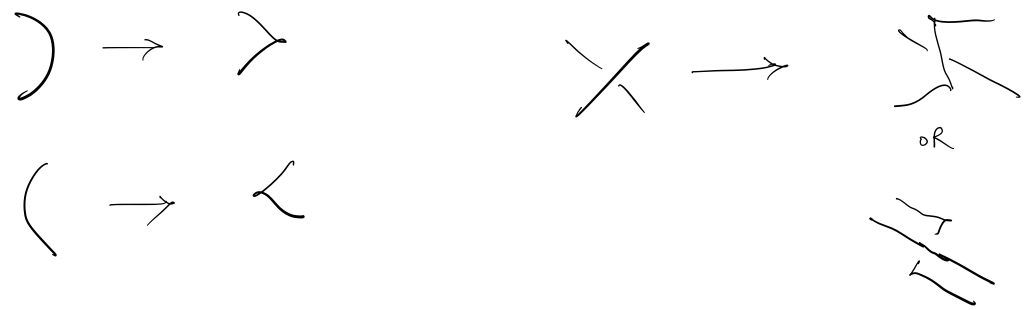


R3



- **Theorem**: Two front projections represent isotopic Legendrian knots $\iff$ they are related via regular homotopy and a sequence of above moves.

- Given a smooth knot, can obtain a Legendrian representative by:



- <u>Invariants</u>:

1) <u>Thurston - Bennequin</u>: $lk(K, K')$ K': pushoff of K.

$$= \text{writhe}(K) - \frac{1}{2}\#\text{cusps}$$

2) <u>Rotation number</u>: # times tangent vector to K points along $\frac{\partial}{\partial y}$.

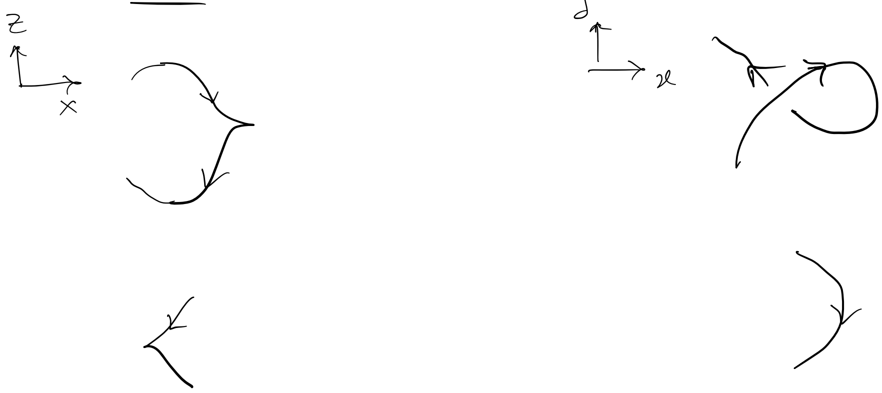
invt of oriented Leg. knot.

$$\text{rot}(K) = \frac{1}{2}\left(\#\,\text{down cusps} - \#\,\text{up cusps}\right)$$

- <u>Lagrangian projection</u> : $(x,y,z) \longrightarrow (x,y)$

  $\rightarrow$ there is a weak Reidemeister-type theorem, no iff condition.

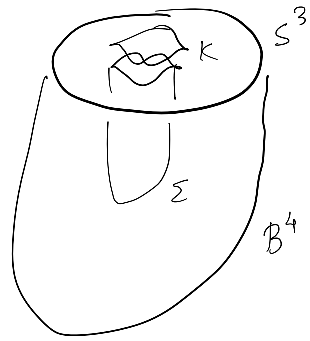
<u>Front</u> <u>Lagrangian</u>



- <u>Turning front projection to Lagrangian</u>

- <u>Double points of Lagrangian projection</u> : Reeb chords

- Fillings of Legendrian knots : Lagrangian surfaces in B^4.



Fact: – These surfaces minimise genus

$$g(\Sigma) = g_4(K).$$

– Need not always exist

Q: Given a knot, how many Lagrangian fillings (upto some notion of isotopy) does it admit?

$LCH(K)$ gives information about this question as well.