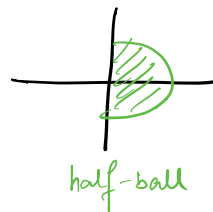
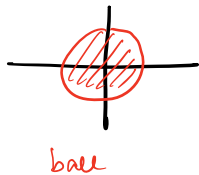
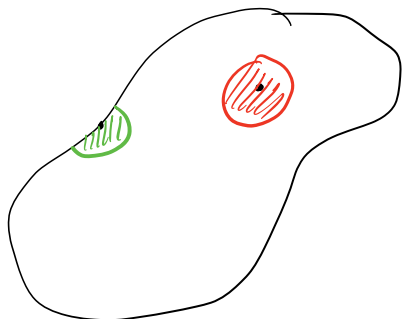
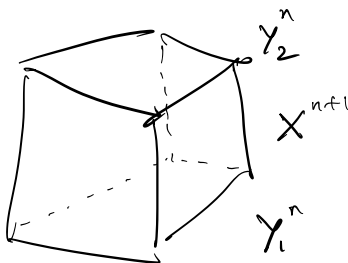


- Manifold with boundary: Every point has a neighbourhood that looks like "ball" or "half-ball" in \mathbb{R}^n .



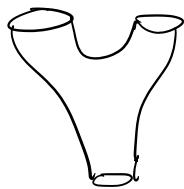
Points with half-ball neighbourhoods are called "boundary" of the manifold.

- Cobordism:



An $(n+1)$ -dim manifold X with boundary $Y_1 \cup Y_2$ is called a cobordism from Y_1 to Y_2 (or Y_2 to Y_1). A direction is assigned to cobordisms.

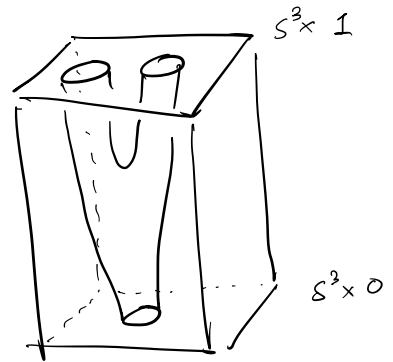
- eg. Pair of pants



cobordism from $\bigcirc \bigcirc$ to \bigcirc

We are interested in:

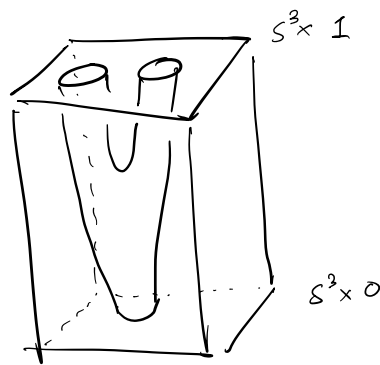
cobordisms living in $S^3 \times [0, 1]$
 for two links in $S^3 \times \{0\}$
 and $S^3 \times \{1\}$.



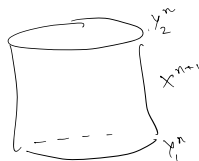
Direction of cobordism: (downwards)

• Filling: A filling of a link $L \subset S^3 \times \{1\}$ is a cobordism to the empty set.

eg.



• General theme:



$n(Y_1)$
invariants

$n(Y_2)$

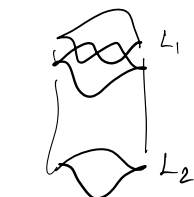
\rightarrow

expect $\phi_x: n(Y_1) \rightarrow n(Y_2)$
 OR vice-versa.

In our case

we have not said what these mean yet

"exact Lagrangian"



cobordisms

given

Legendrian links L_1, L_2 ,

will

give

DGA chain

maps $(d_{L_1}, d_1) \rightarrow (d_{L_2}, d_2)$.

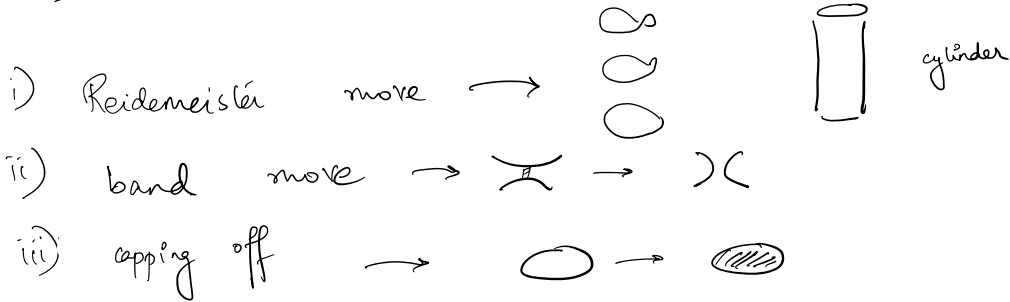
- Fillings will induce DGA chain map to DGA of empty set, which is the same as augmentations.

Thus the general research strategy goes:

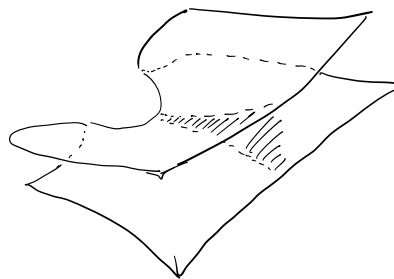
Topology question: What are all the exact Lagrangian fillings of a given Legendrian link? \longleftrightarrow Algebra question: What are all the augmentations of the DGA?

Elementary

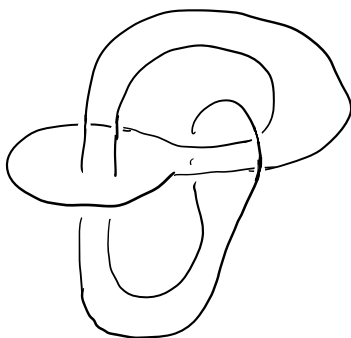
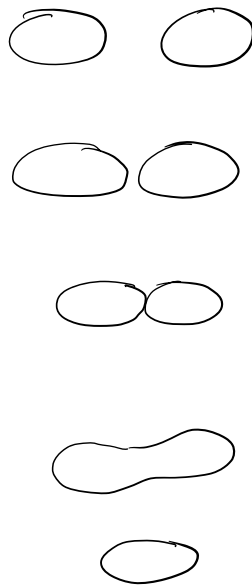
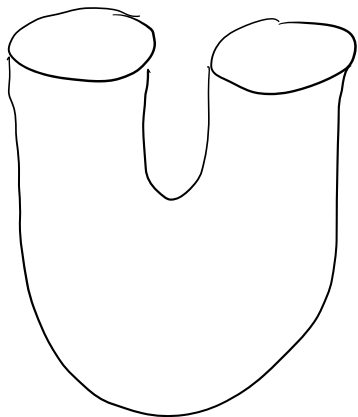
- Cobordisms between knots -



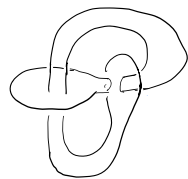
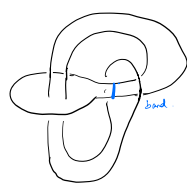
band move:



• Surfaces filling knots in 4-ball (a movie)



→ similar.



R-moves



↓ cap off.

Thus the filling was built from

(Recall

$$2 \text{ disks} + 1 \text{ band} = \text{disk.}$$

$$\chi = \# \text{ disks} - \# \text{ bands} = 1 - 2g$$

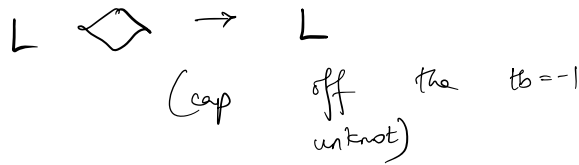
$$\Rightarrow g = 0).$$

• Legendrian knots

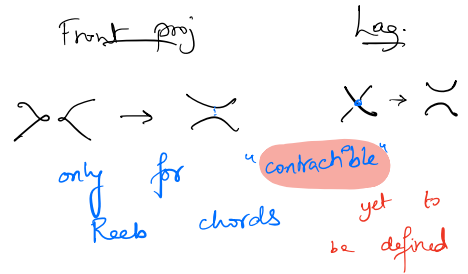
Elementary exact Lagrangian cobordisms:

- 1) Reidemeister moves
- 2) Pinch move (like band move)

- 3) Minimal cobordism

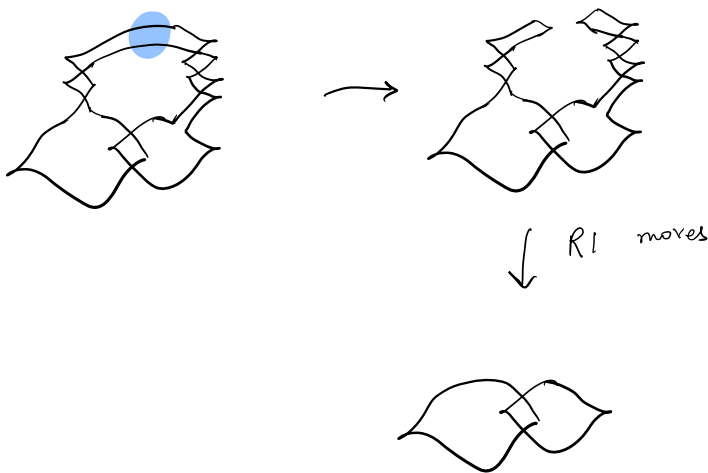


Pinch move

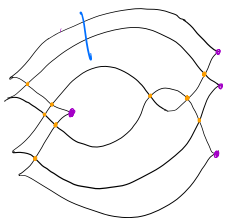


• A decomposable exact Lagrangian cobordism is a concatenation of elementary exact Lagrangian cobordisms.

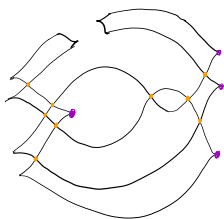
Example: A cobordism to Hopf link



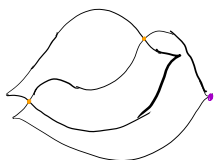
3. Examples:



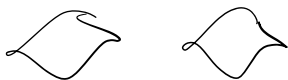
↓ pinch



↓ R1 and R2 moves

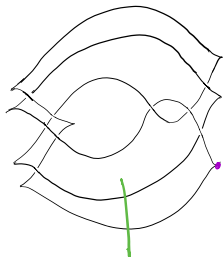


↓ R2



↓ minimal cobordism

2 disks + 1 band
 → disk.



Legendrian knot
 with 2 disk
 fillings.

Exercise