Homework 4

Instructions

Complete the exercises on this page and upload your work to Gradescope by **12:29pm on March 4**.

Be sure to **acknowledge your collaborators**.

Exercises

1. Textbook exercise 5.5.

2. Textbook exercise 5.7.
   Hint: There are probably a lot of ways to do this, but I found it helpful to draw the triangle with vertices P_2, P_4, and P_6. The edge congruences should then allow you to show that $\measuredangle(P_3) = \measuredangle(P_6)$.

3. Textbook exercise 5.9.
   Hint: First show that you can obtain the homeomorphism type of the surface of genus g from a regular $4g$-gon. Using mathematical induction, you can do this by imitating our argument for the surface of genus 2: out of the regular $4g$-gon, cut a domain whose image in the quotient is a punctured torus. Rearrange the remainder of your $4g$-gon into a $4(g-1)$-gon with a hole in its middle and apply an inductive hypothesis.

   Once you have the correct homeomorphism type, you'll need to construct your regular $4g$-gon in $(\mathbb{H}^2, d_{\text{hyp}})$ so that it has the desired angles. Determine these angles, reduce the existence of this regular $4g$-gon to the existence of a certain hyperbolic triangle, and then apply Proposition 5.13.

4. Textbook exercise 5.13.

5. Textbook exercise 5.16.
   Note: This one looks pretty scary, but isn't actually that bad. On part b, pick a vertex of X_i which will serve as a vertex for each of your $n_i - 2$ triangles. On part c, determine how the total angle sum (across all the convex polygons X_i) relates to the number p of images of vertices in $\overline{X}$.

6. Textbook exercise 6.1.
   Hint: Use the triangle inequality on part c. Also, I recommend getting started on this problem early. It shouldn't take that long to do, but I want to make sure we have time to digest the vocabulary.