Homework 2

Instructions

Complete the exercises on this page and upload your work to Gradescope by **12:29pm on February 5**. The optional exercises on page 2 need not be submitted.

Be sure to **acknowledge your collaborators**.

Exercises

1. Textbook exercise 2.7.
2. Textbook exercise 2.9.
3. Textbook exercise 2.10.
4. Textbook exercise 2.15.
5. Textbook exercise 2.16.
6. Textbook exercise 3.5.
7. Textbook exercise 3.6.
8. Let $X = \mathbb{R}^2 - \{(0,0)\}$ be the punctured plane, and let $\overline{X}$ be the partition of X consisting of all rays emanating from the origin. That is, for each $P = (x, y) \in X$, the corresponding element of $\overline{X}$ is the subset
$$\overline{P} = \{(\lambda x, \lambda y) \,|\, \lambda > 0\}.$$
   (a) Identify a subset of $\mathbb{R}^2$ to which $\overline{X}$ is homeomorphic. You don't need to prove the homeomorphism, but describe how you came up with the subset.
   (b) Show that, for every $\overline{P}, \overline{P}' \in \overline{X}$ and for every $\epsilon > 0$, there exist points $Q, Q' \in X$ such that $Q \in \overline{P}$, $Q' \in \overline{P}'$, and $d_{\text{euc}}(Q, Q') < \epsilon$.
   (c) Let $\overline{d}_{\text{euc}}$ be the quotient semi-metric on $\overline{X}$ defined by the Euclidean metric d_{euc} of X. Use the definition of $\overline{d}_{\text{euc}}$ in terms of discrete walks to show that $\overline{d}_{\text{euc}}(\overline{P}, \overline{P}') < \epsilon$ for every $\overline{P}, \overline{P}' \in X$ and for every $\epsilon > 0$.
   (d) Show that $\overline{d}_{\text{euc}}(\overline{P}, \overline{P}') = 0$ for every $\overline{P}, \overline{P}' \in X$. Is $(\overline{X}, \overline{d}_{\text{euc}})$ a metric space?
9. Textbook exercise 4.3.

Optional Exercises

1. Textbook exercise 2.12.
2. Textbook exercise 2.14.
3. Textbook exercise 2.19.
4. Consider the points $P_1 = (0,2)$ and $P_2 = (0,3)$ in the hyperbolic plane $(\mathbb{H}^2, d_{\text{hyp}})$, and let $[P_1, P_2]$ denote the vertical line segment connecting P_1 to P_2.
   (a) For each $P = (0, y)$ in $[P_1, P_2]$ compute the hyperbolic distances $d_{\text{hyp}}(P, P_1)$ and $d_{\text{hyp}}(P, P_2)$.
   (b) Find the hyperbolic midpoint of $[P_1, P_2]$, namely the point $P \in [P_1, P_2]$ such that $d_{\text{hyp}}(P, P_1) = d_{\text{hyp}}(P, P_2)$.
5. Textbook exercise 3.6.
6. In the sphere $\mathbb{S}^2$, let $N = (0,0,1)$ be the north pole. Describe each of the balls

$$B_{d_{\text{sph}}}(N, \tfrac{\pi}{2}), \quad B_{d_{\text{sph}}}(N, \pi), \quad B_{d_{\text{sph}}}(N, \tfrac{3\pi}{2}), \quad \text{and} \quad B_{d_{\text{sph}}}(N, 2\pi)$$

   with a picture and a few words.
7. Textbook exercise 4.1.
8. Textbook exercise 4.4.