Math 4803 February 14, 2024

LAST TIME

Our first examples of

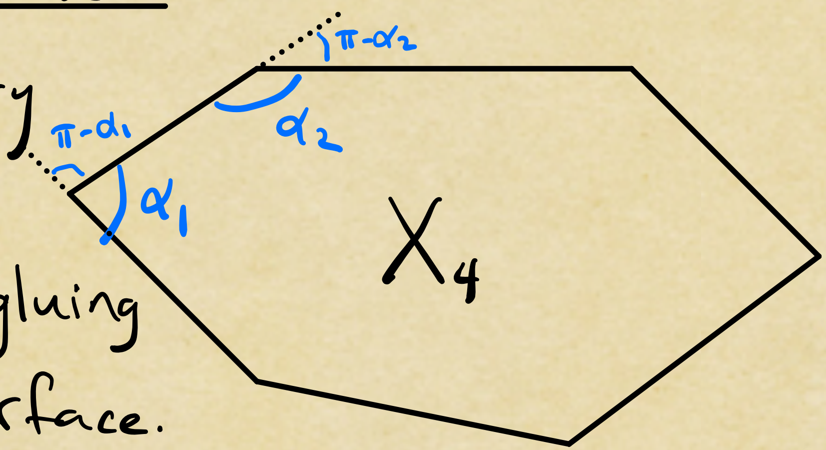
Euclidean surfaces.

TODAY

Surfaces of all three types.

Edge gluings of hexagons

Suppose that X_4 is an arbitrary convex hexagon in $(\mathbb{R}^2, d_{euc})$ on which we want an edge gluing that yields a Euclidean surface.



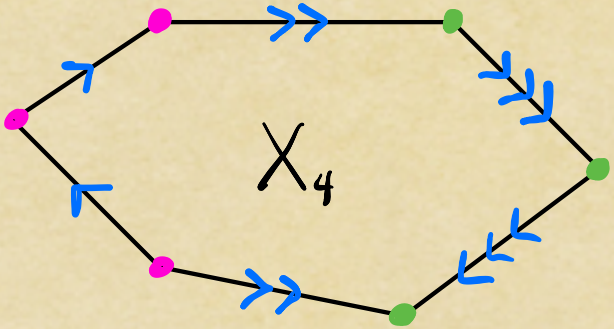
The interior angles $\alpha_1, \ldots, \alpha_6$ sum to $\underline{4\pi}$, since

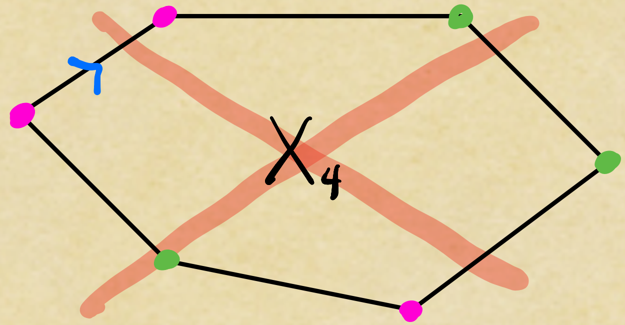
$$6\pi - \sum_{i=1}^{6} \alpha_i = \sum_{i=1}^{6} (\pi - \alpha_i) = 2\pi.$$

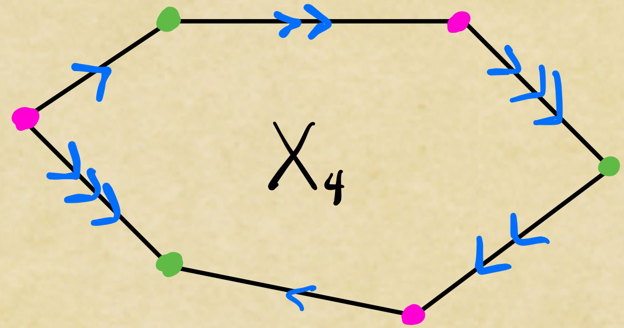
From this we learn that the vertices $P_1, \ldots, P_6$ must be divided by our edge gluing into $\underline{\text{exactly two}}$ equivalence classes, $\underline{\text{each of size three}}$. This is because $\alpha_i \in (0, \pi)$ and $\sum_{Q \in \overline{P}} \measuredangle(Q) = 2\pi$.

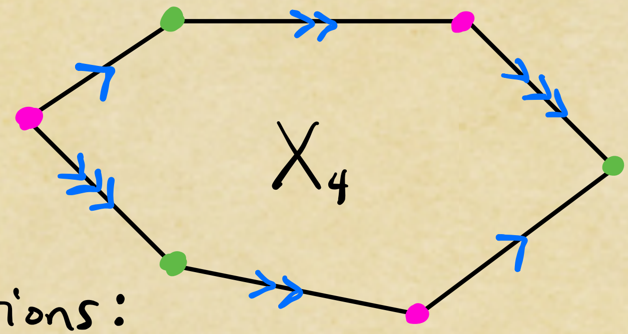
Edge gluings of hexagons

This leaves three possibilities:

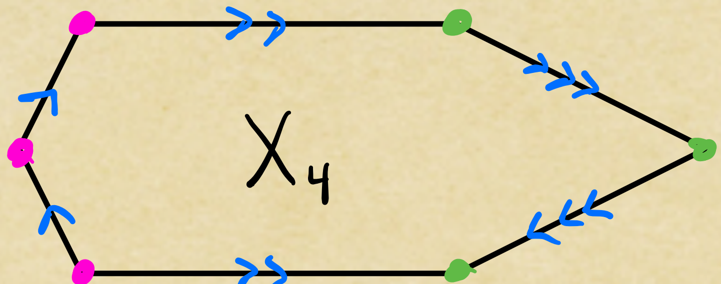


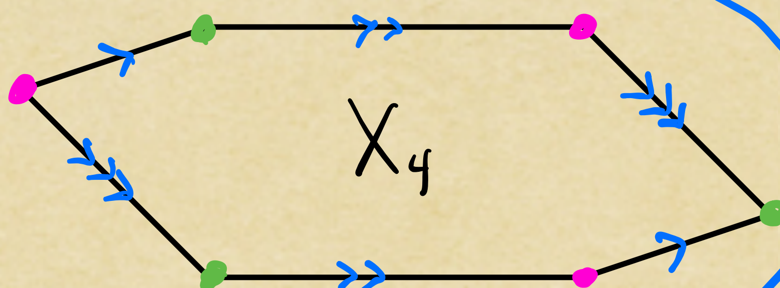






Edge decorations ~~cut these to two~~ leave us with 3 options:

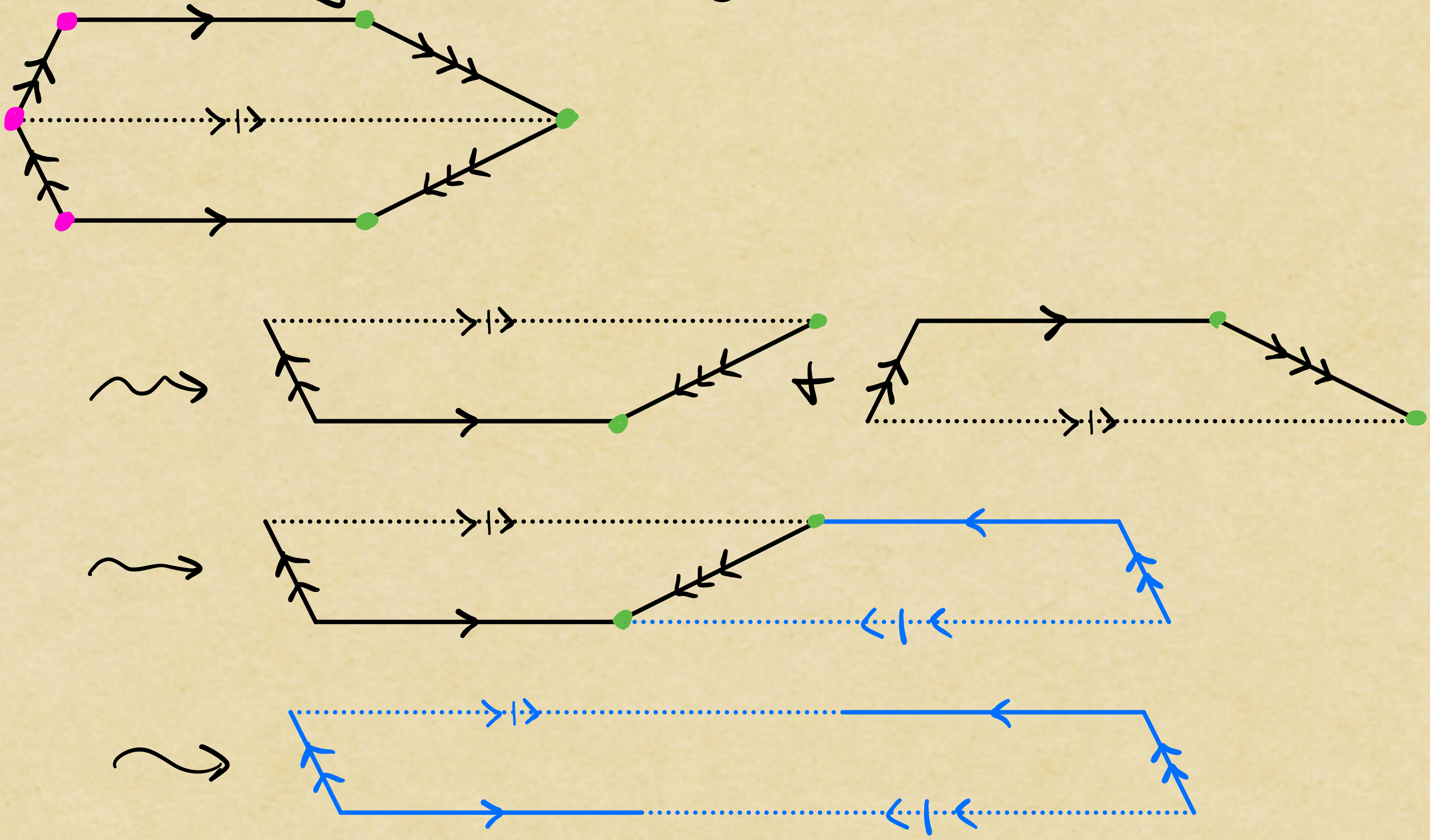




Let's determine what these give us.

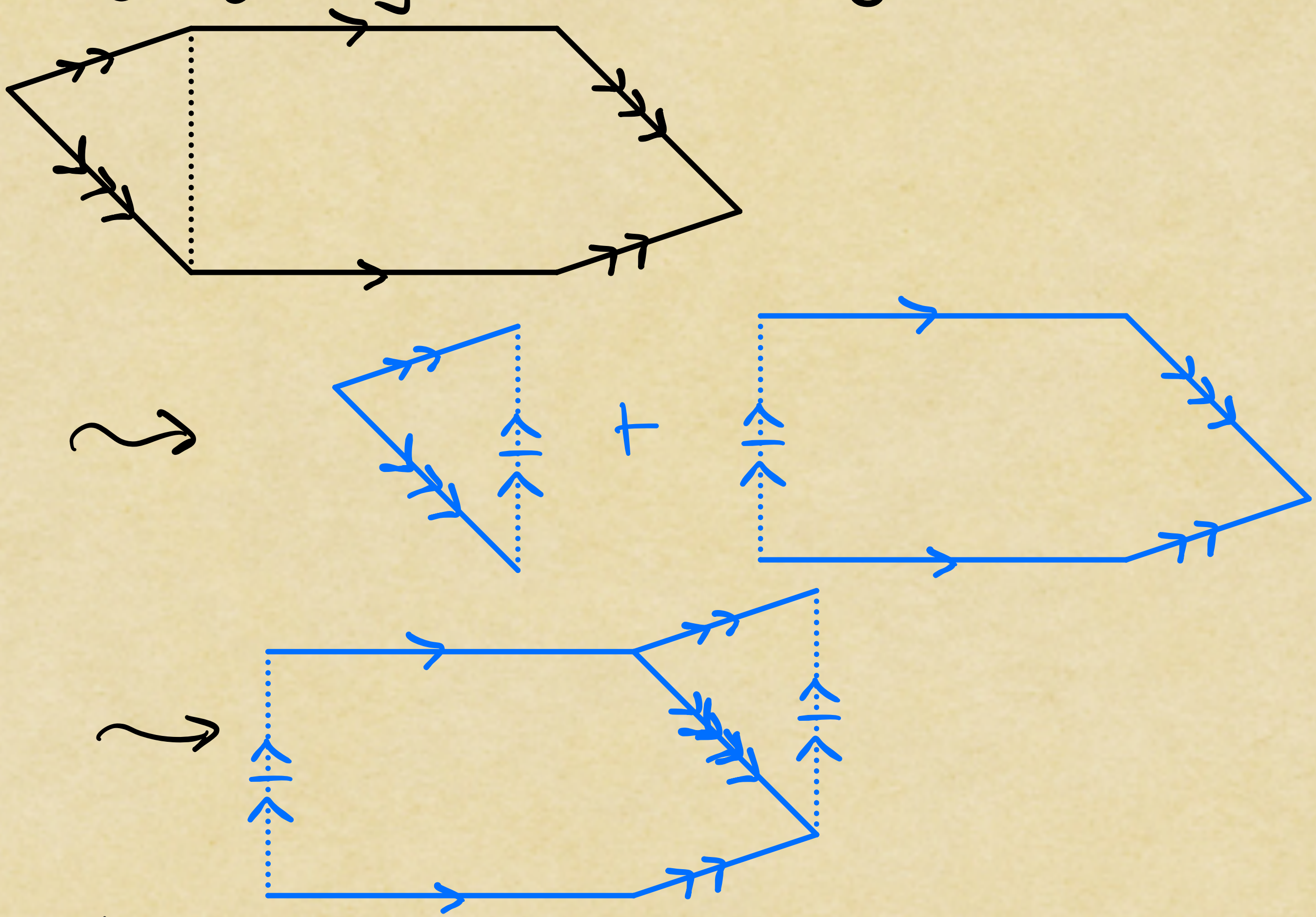
I omitted the one in the top right when I prepped.

Edge gluings of hexagons



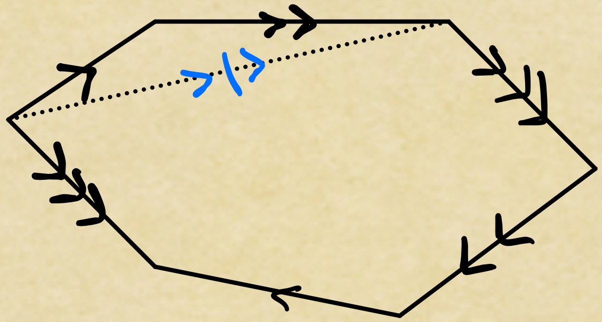
This gives a Klein bottle.

Edge gluings of hexagons

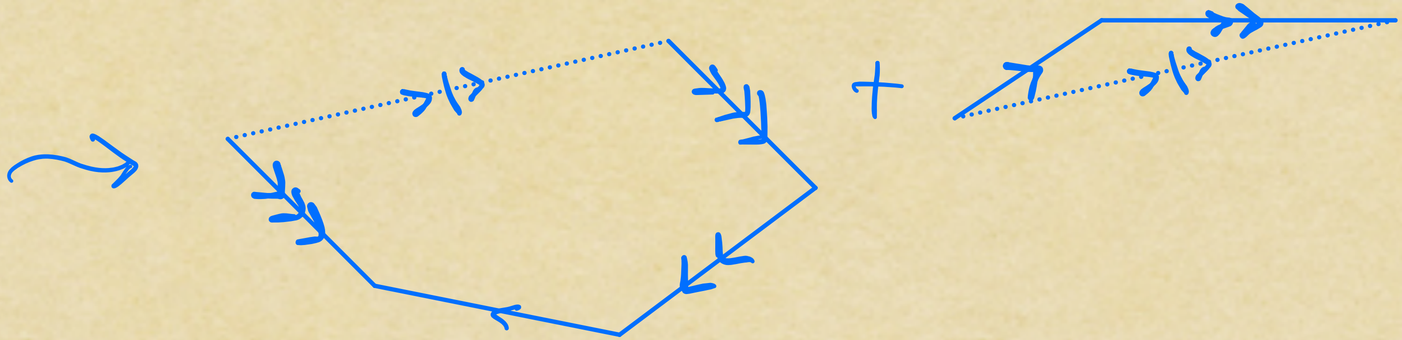


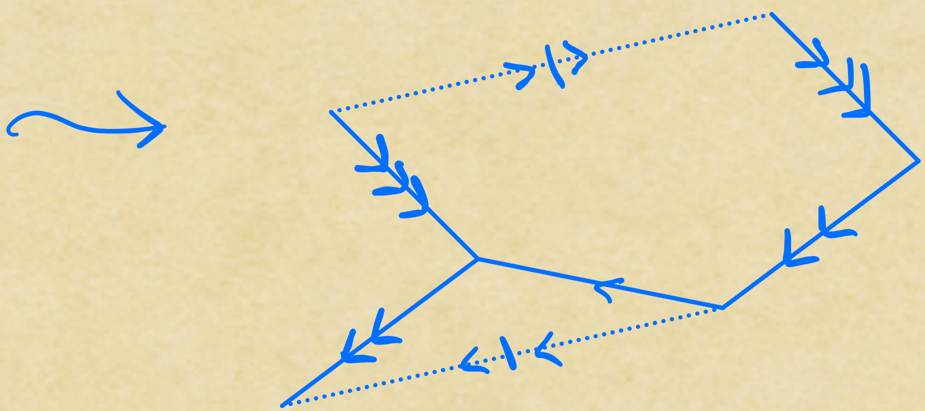
This gives a Euclidean surface homeomorphic to a torus, but not isometric to a torus obtained from a parallelogram!

Edge gluings of hexagons



(Not in original notes b/c I missed it.)

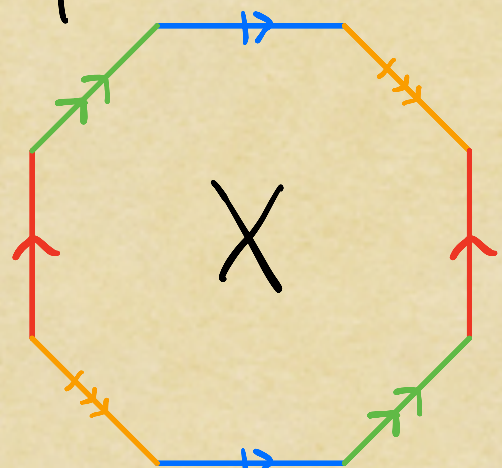




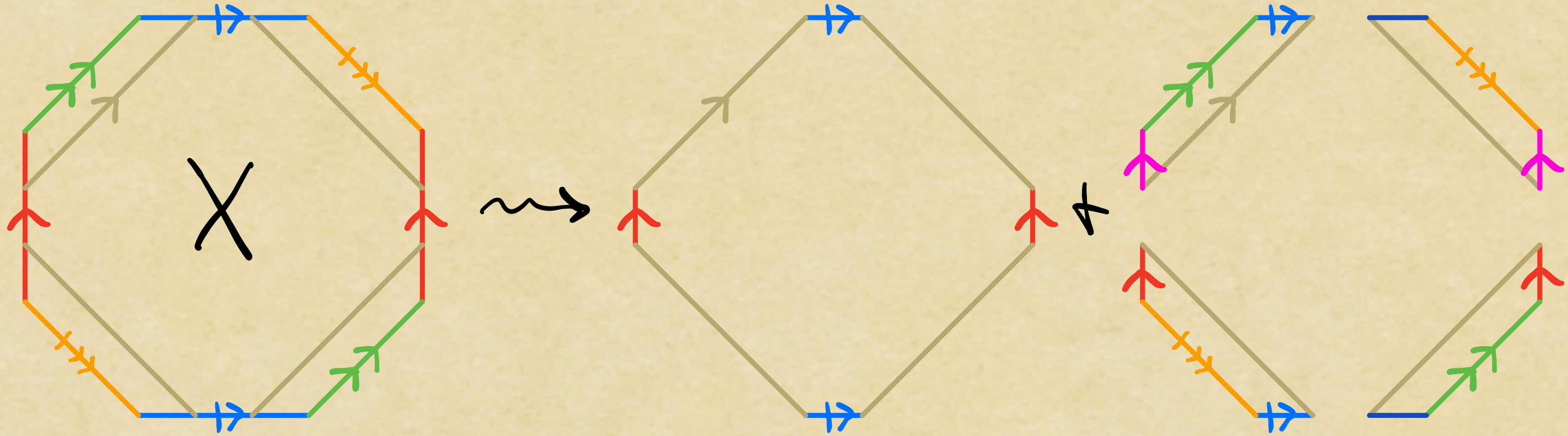
Another Klein bottle!

The surface of genus two

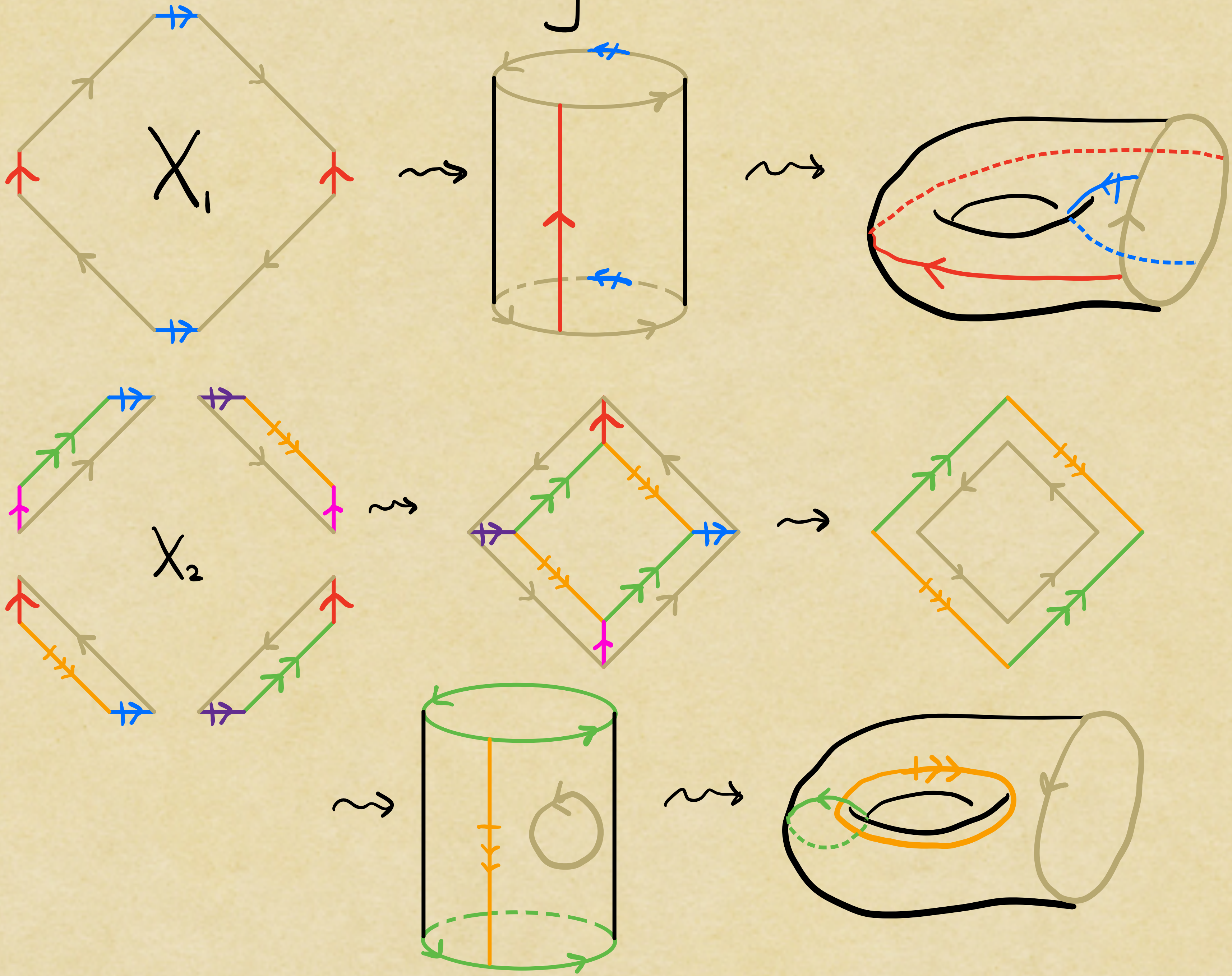
Let's think about the homeomorphism type of the metric space obtained from the following edge gluing:



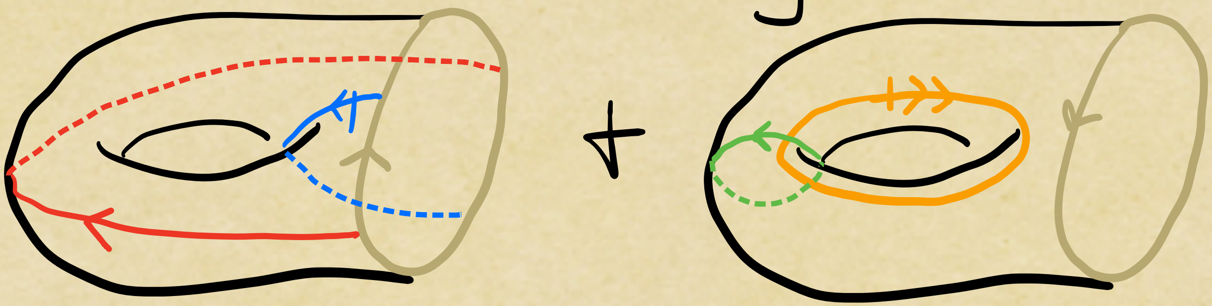
We'll momentarily ignore the geometry of this space.

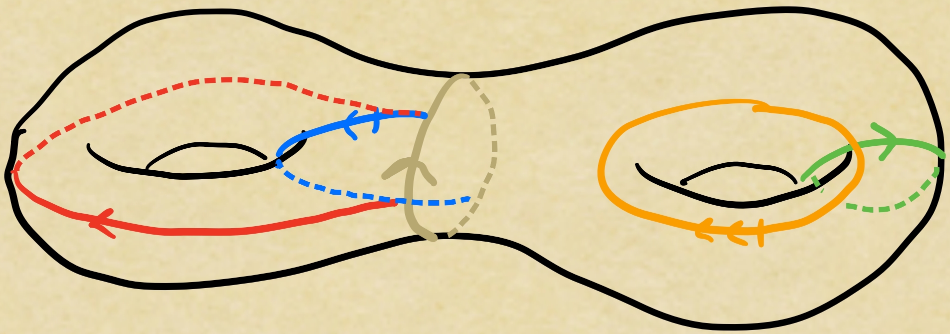


The surface of genus two

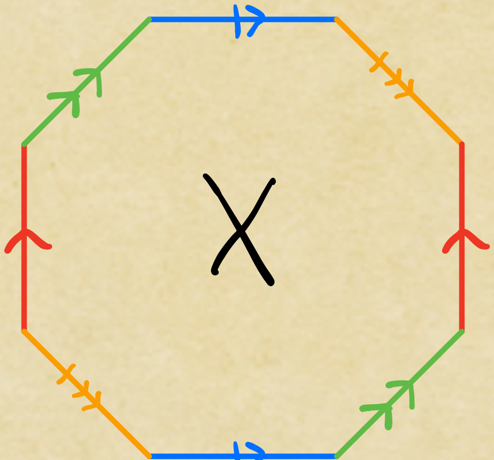


The surface of genus two





Now what about the geometry?



All 8 vertices are equivalent under this edge gluing. Since the angle sum is $6\pi \neq 2\pi$, we don't get a Euclidean surface.