

Math 4441: Differential Geometry
Georgia Institute of Technology
Fall 2022

Lecture Information

Instructor: Austin Christian

Office: Skiles 022

Meeting location: College of Computing 52
Zoom (see Canvas)

Meeting time: MWF 9:30am-10:20am, and via recording.

Email: austin.christian@math.gatech.edu

Office Hours: See website, and by request.

General Information

Catalog description

The theory of curves, surfaces, and more generally, manifolds. Curvature, parallel transport, covariant differentiation, Gauss-Bonnet theorem.

Pre-/co-requisites

Formally, you need one of the following MATH credits with a satisfactory grade: 2401, 2411, 24X1, 2605, 2551, 2561, 2550, or 2X51. Informally, this class will rely heavily on your knowledge of linear algebra and multivariable calculus, and require comfort with reading and writing mathematical proofs.

Course goals and learning outcomes

At the conclusion of this course, the student should be able to:

- compute the Frenet-Serret apparatus for a regular curve in \mathbb{R}^3 ;
- calculate the first and second fundamental forms of a smooth surface in \mathbb{R}^3 , and use these to produce the principal curvatures of the surface;
- define parallel transport along regular curves and explain the notion of a geodesic curve along a smooth surface in \mathbb{R}^3 ;
- differentiate between intrinsic and extrinsic properties of smooth surfaces in \mathbb{R}^3 ;
- summarize and employ major results such as Gauss' *Theorema Egregium* and the Gauss-Bonnet theorem;
- compose an original, expository narrative focused on a topic intersecting with geometry or topology, including proofs and examples.

Course Requirements and Grading

There will be 12 in-class activities, 2 midterms (in class), approximately 6 homework assignments, and a term paper, all to be submitted via Gradescope. These components will be explained in more detail below; the most important dates are summarized in Table 1. Homework due dates will be announced in class, and you'll typically have about two weeks to complete each assignment.

Active learning

Approximately once per week, course content will be delivered via an activity instead of a lecture. The activity will include several exercises, with new material introduced in and between the exercises. You are responsible for this material, just as if it were delivered in a lecture.

The activities can be completed individually or in groups of up to three students, and attendance on active learning days will account for 20% of the grade for the corresponding activity. The activities are designed to be completed in class, but you'll have about five days to write up your work for submission.

Assessment due	Date	Time
Activity 1	8/31 (Wed)	11:59pm
Activity 2	9/7 (Wed)	11:59pm
Activity 3	9/14 (Wed)	11:59pm
Activity 4	9/21 (Wed)	11:59pm
Activity 5	9/28 (Wed)	11:59pm
Midterm 1	9/30 (Fri)	in class
Term paper consult	9/30 (Fri)	5:00pm
Activity 6	10/12 (Wed)	11:59pm
Activity 7	10/19 (Wed)	11:59pm
Activity 8	10/26 (Wed)	11:59pm
Activity 9	11/2 (Wed)	11:59pm
Activity 10	11/9 (Wed)	11:59pm
Activity 11	11/16 (Wed)	11:59pm
Term paper draft	11/18 (Fri)	11:59pm
Midterm 2	11/21 (Mon)	in class
Term paper feedback	11/28 (Mon)	11:59pm
Activity 12	12/7 (Wed)	11:59pm
Term paper	12/9 (Fri)	11:59pm

Table 1: Fixed due dates

Term paper

In lieu of a final exam, our course will culminate with a term paper, written individually or in a group of up to three students. The 4–8 page expository paper will report on some topic in differential geometry or topology (or a related field) and will be graded both on its mathematical content and its exposition. A more detailed rubric, as well as topic suggestions, will be provided in class and through the course webpage.

You will choose between two grading schemes for your paper. The schemes are given below, and you must make your selection by **October 14**. Once you select your scheme, it can only be changed under extraordinary circumstances.

Requirement	Scheme 1	Scheme 2
consult by Sept 30	5%	5%
draft submitted by Nov 18	15%	15%
constructive feedback by Nov 28	20%	20%
final paper by Dec 9	60%	55%
art submission by Dec 9	N/A	10%

Notice that Scheme 2 allows you to earn 105%. In this scheme, you would create a work of art (broadly construed) corresponding to the topic of your paper. The parameters for this artwork will be quite loose, but you should consult with me by **Nov 18** about the acceptability of your submission. **This is the only opportunity for extra credit in this course.**

In both schemes, you will need to consult with me by September 30 so that I can approve your topic. You will also provide constructive feedback on other papers, and your feedback will be graded. More details will be available in class and through the course webpage.

Collaborations

You are encouraged to collaborate on homework, active learning, and your term paper — indeed, the latter two can be submitted by a group rather than an individual. However, in all cases you should generously acknowledge

the contributions of others.

This concern is most acute on the homework assignments. You should write up all solutions by yourself, and acknowledge any sources from which you received help. This could be other students, textbooks, or the internet. Submitting the work of others as your own (even if you've, say, reworded or paraphrased) will be treated as academic misconduct. On the other hand, substantial partial credit can be awarded for writing something like, "I was unable to figure this problem out, but here's a solution I read online ([<url>](#)), which I've tried to put in my own words."

Final grade

Your course average will be computed as follows:

$$20\% \text{ homework} + 30\% \text{ in-class activities} + 10\% \text{ midterm 1} + 10\% \text{ midterm 2} + 30\% \text{ term paper}$$

Your final grade will be assigned as a letter grade. The scale will be **no more harsh than:**

A	B	C	D	F
≥ 90	[80, 90)	[70, 80)	[60, 70)	< 60

Note that this scale does not imply any rounding.

Course Materials and Technologies

Course text

Our main text will be Michael Willis' lecture notes:

<https://www.math.ucla.edu/~mike.willis/ClassText.pdf>

We will draw material from other sources, but follow the notation of the above notes as much as possible. Some sources include:

- *Differential Geometry of Curves and Surfaces*, do Carmo
- *Elementary Differential Geometry*, O'Neill
- *Elements of Differential Geometry*, Millman and Parker
- *Differential Geometry: A First Course in Curves and Surfaces*, Shifrin (<http://alpha.math.uga.edu/~shifrin/ShifrinDiffGeo.pdf>)
- Prof. Ghomi's lecture notes (<https://ghomi.math.gatech.edu/LectureNotes>)
- *Classical Differential Geometry*, Petersen (<https://www.math.ucla.edu/~petersen/DiffGeo.pdf>)
- *Experimental Notes on Elementary Differential Geometry*, Yang (<https://deaneyang.github.io/DifferentialGeometry/DifferentialGeometry.pdf>)

Many of these texts are available at little or no cost, and I will try to let you know which topics are coming from which sources. Please get in touch if you'd like more detailed recommendations.

Course websites

- The class webpage (<https://austinchristian.math.gatech.edu/teaching/4441-f22/>) is where I will post class notes, active learning assignments, and homework assignments. This page will also have a regularly-updated course schedule.
- Canvas (<https://gatech.instructure.com/courses/264070>) will be used for the gradebook, announcements, and class recordings.
- Gradescope (accessed through Canvas) will be used to collect and return graded assessments. This is also where you can submit regrade requests.

- Discord (<https://discord.gg/aTuba9vAwa>). I've created a Discord server for our class, and I prefer to discuss math here rather than via email. This will lead to quicker responses, and reduces duplicate questions. **Note:** Please apply common sense to your use of our Discord server. Do not use the server during an exam, do not post illegal or inappropriate content, and do not post commercial advertisements.

Mathematica

For some of our active learning assignments, a *Mathematica* notebook will be made available to help with some of the computations and visualizations. As a Georgia Tech student, you have access to *Mathematica* through the Office of Information Technology. You should request this software (via <https://www.software.oit.gatech.edu>) as soon as possible, and begin getting comfortable with programming in *Mathematica*. As much as possible, the notebooks will be designed to be accessible without previous familiarity with *Mathematica*.

Email protocol

Our plan is to not have math discussions by email this semester. Math questions should be posted on Discord, so that everyone in the class has a chance to follow the discussion. (Feel free to ask your questions anonymously.) Any math questions emailed to me will be redirected to Discord.

Course Expectations and Guidelines

Academic integrity

Georgia Tech aims to cultivate a community based on trust, academic integrity, and honor. Students are expected to act according to the highest ethical standards. For information on Georgia Tech's Academic Honor Code, please visit <http://www.catalog.gatech.edu/policies/honor-code/> or <http://www.catalog.gatech.edu/rules/18/>.

Accommodations for students with disabilities

If you are a student with learning needs that require special accommodation, contact the Office of Disability Services at (404)894-2563 or <http://disabilityservices.gatech.edu/>, as soon as possible, to make an appointment to discuss your special needs and to obtain an accommodations letter. Please also email me as soon as possible in order to set up a time to discuss your learning needs.

Lecture format and attendance

All lectures will be delivered in three formats: in person, synchronously on Zoom, and via recording, to be made available on Canvas. Synchronous attendance is not mandatory, but is **very strongly** encouraged. Active learning days will not be recorded, but can be made available on Zoom when necessary. As mentioned above, synchronous attendance on active learning days is mandatory.

Class etiquette

Whether attending class in-person or online, please be respectful of your classmates and instructor. In person, please refrain from personal conversations and any behavior that might be distracting to your classmates or instructor. Online, feel free to use the chat feature to ask relevant questions, but refrain from posting inappropriate content or engaging in unrelated discussions.

We will maintain a respectful, inclusive classroom culture, and **disrespect of your classmates or instructor will not be tolerated.**

Regrade requests

Any regrade request should be submitted on Gradescope, with an explanation of the reason, within one week of the date the graded exam has been returned to you. **Papers submitted for regrading could be adjusted up or down;** so please make sure to check the solutions before submitting a regrade request.

Student-faculty expectations agreement

At Georgia Tech we believe that it is important to strive for an atmosphere of mutual respect, acknowledgement, and responsibility between faculty members and the student body. See <http://www.catalog.gatech.edu/rules/22/> for an articulation of some basic expectation that you can have of me and that I have of you. In the end, simple respect for knowledge, hard work, and cordial interactions will help build the environment we seek. Therefore, I encourage you to remain committed to the ideals of Georgia Tech while in this class.

Campus Resources for Students

In your time at Georgia Tech, you may find yourself in need of support. Below you will find some resources to support you both as a student and as a person.

Academic support

- Drop-In Tutoring/Math Lab: <https://tutoring.gatech.edu/drop-in/>
 - The Math Lab offers free drop-in math help, and is staffed by math Graduate Teaching Assistants.
 - A live schedule for the Math Lab can always be found at the above website, and any questions can be directed to dropintutoring@gatech.edu.
- Center for Academic Success: <http://success.gatech.edu>
 - 1-to-1 tutoring: <http://success.gatech.edu/1-1-tutoring>
 - Peer-Led Undergraduate Study (PLUS): <http://success.gatech.edu/tutoring/plus>
 - Academic coaching: <http://success.gatech.edu/coaching>
- Residence Life's Learning Assistance Program: <https://housing.gatech.edu/learning-assistance-program>
- OMED Educational Services: <http://omed.gatech.edu/programs/academic-support>
- Communication Center: <http://www.communicationcenter.gatech.edu>
- Academic advisors for your major: <http://advising.gatech.edu/>

Personal support

- The Office of the Dean of Students: <https://studentlife.gatech.edu/>; 404-894-6367; Smithgall Student Services Building 2nd floor. You also may request assistance at https://gatech-advocate.symplicity.com/care_report/index.php/pid383662?
- Counseling Center: <http://counseling.gatech.edu>; 404-894-2575; Smithgall Student Services Building 2nd floor.
 - Services include short-term individual counseling, group counseling, couples counseling, testing and assessment, referral services, and crisis intervention. Their website also includes links to state and national resources.
 - Students in crisis may walk in during business hours (8am-5pm, Monday through Friday) or contact the counselor on call after hours at 404-894-2204.
- Students' Temporary Assistance and Resources (STAR): <https://studentlife.gatech.edu/content/star-services>. Can assist with interview clothing, food, and housing needs.
- Stamps Health Services: <https://health.gatech.edu>; 404-894-1420. Primary care, pharmacy, women's health, psychiatry, immunization and allergy, health promotion, and nutrition.
- Women's Resource Center: <https://www.womenscenter.gatech.edu>; 404-385-0230.
- LGBTQIA Resource Center: <https://lgbtqia.gatech.edu>; 404-385-2679.

- Veteran's Resource Center: <https://veterans.gatech.edu>; 404-385-2067.
- Georgia Tech Police: 404-894-2500.

Statement of Inclusivity

As a member of the Georgia Tech community, I am committed to creating a learning environment in which all of my students feel safe and included. Because we are individuals with varying needs, I am reliant on your feedback to achieve this goal. To that end, I invite you to enter into dialogue with me about the things I can stop, start, and continue doing to make my classroom an environment in which every student feels valued and can engage actively in our learning community.

A note from Austin

It is important to me that you not become overwhelmed in this class. Mathematics can be very challenging and is often frustrating, but you shouldn't feel that succeeding in mathematics is impossible. **Everyone is a math person.** If you feel that the coursework is beginning to slip away from you, please let me know *before* you become completely lost. I expect you to invest a lot of time and energy into this course, but I am committed to helping you learn and enjoy the material, and will do my best to help you succeed.

Changes to the syllabus

This syllabus represents my expectations for the content and timing of this course as well as possible. However, because these expectations may change, I reserve the right to modify course policies as the need arises. If this happens, students will be notified by email and in class.