

Math 4441

September 26, 2022

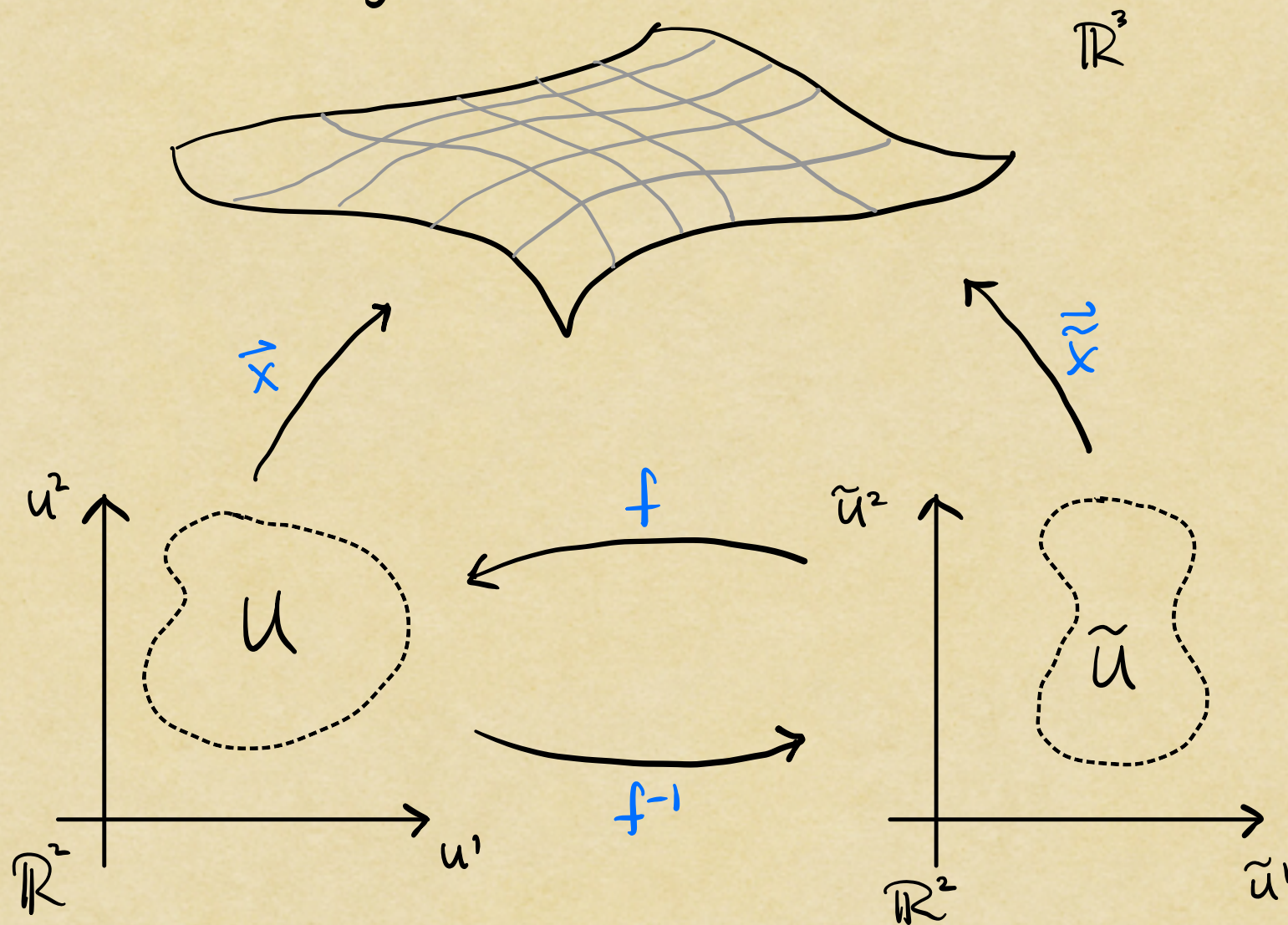
What's going on?

We reinterpreted multivariable differentiation so that we get a linear map at each point of our domain — i.e., a linear approx.

We then defined simple surfaces in a way that felt like "curves with 2D domains".

In this analogy, coordinate transformations should play the role of reparametrizations.

Schematically:



Lemma If $\vec{x}: U \rightarrow \mathbb{R}^3$ is a C^k simple surface and $f: \tilde{U} \rightarrow U$ is a C^k coordinate transformation, then

$$\tilde{\vec{x}} := \vec{x} \circ f$$

is also a C^k simple surface, with the same image as $\vec{x}$.

(Proof.) Since f is surjective, we immediately have $\text{im}(\tilde{\vec{x}}) = \text{im}(\vec{x})$.

Next, we need $\tilde{\vec{x}}$ to be injective and C^k .

These properties follow from the fact that $\vec{x}$
and f are injective $\vdash C^k$ ($\vdash$ chain rule).

Finally, we need regularity. From the chain rule,

$$d\vec{x}_p = d\vec{x}_{f(p)} \cdot df_p$$

Suppose $d\vec{x}_p(\vec{v}) = \vec{0}$, for some $\vec{v}$. Then

$$d\vec{x}_{f(p)}(df_p(\vec{v})) = \vec{0} \Rightarrow df_p(\vec{v}) = \vec{0},$$

because $\ker d\vec{x}_{f(p)} = \{\vec{0}\}$ But $\det(df_p) \neq 0$,

So we must have $\vec{v} = \vec{0}$.



Next time

Our first geometric invariants of simple surfaces: the unit normal vector and the tangent plane.