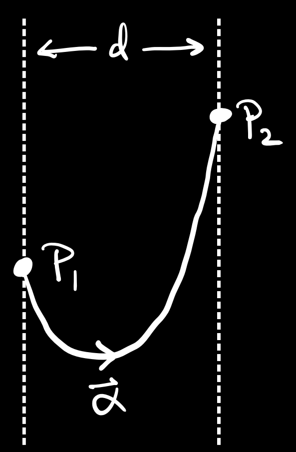
A sketchy argument for #6 on HW 2.

**Note: If you use this, fill in details and write in your own words/ figures!**

Choose points $P_1 = \vec{\alpha}(s_1)$ & $P_2 = \vec{\alpha}(s_2)$ s.t. the tangent lines $\ell_{P_1}, \ell_{P_2}$ are parallel.




Up to isometry, $\ell_{P_1}$ & $\ell_{P_2}$ are vertical. Assume $s_1 < s_2$. (And that $\vec{\alpha}$ is unit-speed)

Let $d$ be the distance between $\ell_{P_1}$ & $\ell_{P_2}$.

Claim ① With this setup, $d = \int_{s_1}^{s_2} \langle \vec{\alpha}'(s), \vec{e}_1 \rangle \, ds$.

(Proof.) For you to do. ▨

Claim ② In fact, $d = \int_{s_1}^{s_2} \cos\theta(s) \, ds$, where $\theta(s)$ is the angular rotation function.

(Proof.) Still for you, but follows from Claim ①. ▨

Claim 3 The function $\theta : [s_1, s_2] \to [-\pi/2, \pi/2]$ is ~~bijective~~ surjective (but might reverse order).

(Proof.) Hint: $\theta' = k$. ∎

Based on 2, we have

$$d = \int_{s_1}^{s_2} \cos\theta(s)\, ds = \frac{1}{\kappa_0} \int_{s_1}^{s_2} \kappa_0 \cos\theta(s)\, ds$$

(use words to justify these steps)

$$\geq \frac{1}{\kappa_0} \int_{s_1}^{s_2} k(s) \cos\theta(s)\, ds$$

$$= \frac{1}{\kappa_0} \int_{s_1}^{s_2} \theta'(s) \cos\theta(s)\, ds.$$

Now apply a u-sub with $u = \theta(s)$. (Thanks, Matthew!) Be careful with bounds. (It may be easiest to assume that $k \geq 0$, but explain why you can do this.) Eventually you'll see an integral you can do.

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Thanks to Parul, Ethan, Izah, Matthew for discussing this with me!

Let me know of other solutions you find!