Math 2552 Extra Problems
Fall 2021/Week 9

Problems

1. Find the general solution for each of the following ODEs.
   (a) $y''' - y' = 0$
   (b) $y''' - y' = 60e^{4t} - 2e^{t}$
   (c) $y''' + 4y'' + 13y' - 50y = 0$
   (d) $y''' + 4y'' + 13y' - 50y = -2190\cos(4t)$
   (e) $y^{(4)} - y = 0$

2. Solve each of the following IVPs.
   (a) $y'' - 4y' + 4y = 14e^{2t}$, $y(0) = 5$, $y'(0) = 7$
   (b) $y'' + 6y' + 25y = 8e^{-3t}\sin(4t)$, $y(0) = 3$, $y'(0) = -30$
   (c) $y'' + 6y' + 9y = (12t^2 - 6)e^{-3t}$, $y(0) = 4$, $y'(0) = -7$
   Pay special attention to this last one.

Answers

1. (a) $y(t) = c_1e^{-t} + c_2 + c_3e^t$

   (b) $y(t) = c_1e^{-t} + c_2 + c_3e^t + e^{4t} - te^t$

   (c) $y(t) = e^{-3t}(c_1\cos(4t) + c_2\sin(4t))$

   (d) $y(t) = e^{-3t}(c_1\cos(4t) + c_2\sin(4t)) + 19\cos(4t) + 2\sin(4t)$

   (e) $y(t) = c_1\cos(t) + c_2\sin(t) + c_3e^{-t} + c_4e^t$

2. (a) $y(t) = e^{2t}(7t^2 - 3t + 5)$

   (b) $y(t) = e^{-3t}(3\cos(4t) - 5\sin(4t)) - te^{-3t}\cos(4t)$

   (c) $y(t) = 4e^{-3t} + 5te^{-3t} + t^2(t^2 - 3)e^{-3t}$
   The special attention to be paid here is the following: the homogeneous solutions are e^{-3t} and te^{-3t}, while the forcing term suggests a particular solution of the form $y_p(t) = (At^2 + Bt + C)e^{-3t}$. Since there's a t^2 involved, this is not a homogeneous solution. But *parts* of this proposed $y_p(t)$ are homogeneous solutions (namely, the Bte^{-3t} and Ce^{-3t} parts), so we multiply the whole thing by t until no parts are homogeneous solutions. So our particular solution has the form $y_p(t) = t^2(At^2 + Bt + C)e^{-3t}$.