

Problems

1. Find a general solution for each of the following ODEs.

(a) $y'' - 4y' - 21y = 6 \cos(3t) - 72 \sin(3t)$

(b) $y'' - 9y = 16 \cos(2t)e^{4t} + 3 \sin(2t)e^{4t}$

(c) $y'' - 6y' + 10y = 6 \cos(2t) + 12 \sin(2t)$

(d) $y'' - 6y' + 9y = -8e^{5t} \sin(2t)$

(e) $y'' - 6y' = (57 - 9t) \cos(3t) + (135 - 27t) \sin(3t)$

(There was previously a sign error in this last ODE which made it messier than intended :(But it was still solvable.)

2. Solve each of the following IVPs.

(a) $y'' + y' = 4 \cos(4t) - 16 \sin(4t), y(0) = -1, y'(0) = 9$

(b) $y'' + 4y' + 4y = 16e^{2t}, y(0) = 2, y'(0) = 4$

(c) $y'' - 4y = -2e^t(2 \cos(t) + \sin(t)), y(0) = 6, y'(0) = 7$

(d) $y'' + y = (2 - 3t^2) \cos(2t) - 8t \sin(2t), y(0) = 1, y'(0) = 4$

(e) $y'' + 2y' + 5y = e^t(4 + 7t) \cos(t) - 2e^t(1 + 2t) \sin(t), y(0) = 2, y'(0) = -7$

Answers

1. (a) $y(t) = c_1 e^{-3t} + c_2 e^{7t} - \cos(3t) + 2 \sin(3t)$
(b) $y(t) = c_1 e^{-3t} + c_2 e^{3t} + e^{4t} \sin(2t)$
(c) $y(t) = e^{3t}(c_1 \cos(t) + c_2 \sin(t)) + \cos(2t)$
(d) $y(t) = c_1 e^{3t} + c_2 t e^{3t} + e^{5t} \cos(2t)$
(e) $y(t) = c_1 + c_2 e^{6t} + (t - 5)(\sin(3t) - \cos(3t))$
2. (a) $y(t) = 4 - 5e^{-t} + \sin(4t)$
(b) $y(t) = e^{-2t} + 4te^{-2t} + e^{2t}$
(c) $y(t) = e^{-2t} + 4e^{2t} + e^t \cos(t)$
(d) $y(t) = \cos(t) + 4 \sin(t) + t^2 \cos(2t)$
(e) $y(t) = e^{-t}(2 \cos(2t) - 3 \sin(2t)) + t e^t \cos(t)$

Note: These are answers, not solutions. Be sure you know the difference.