

Problems

1. Consider the ODE $y'' - 4y' + 4y = 0$. (Note: This ODE previously had a typo. Thanks to a classmate for catching it!)
 - (a) Let $x = y'$ and rewrite the ODE so that it has order one.
 - (b) We now have a linear system of first-order ODEs. Put it in matrix form and find the general solution of the original ODE.
2. For each of the following linear systems, find all equilibria, find the general solution, sketch the phase portrait, and determine the stability of each equilibrium.

(a)
$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 3 & -3 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} x_1 - 2 \\ x_2 - 3 \end{pmatrix}$$

(b)
$$\vec{x}' = \begin{pmatrix} 0 & 5 \\ -2 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} -5 \\ -2 \end{pmatrix}$$

(c)
$$\vec{x}' = \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$