Math 2552
Semester Outline

Here's a high-level outline of what we learned this semester. It's not exactly a study guide, just a way to organize your thoughts about what we did.

1 Introduction to differential equations

- Why do we study differential equations?
- Ordinary versus partial differential equations; linear versus non-linear; autonomous ODEs
- Order of an ODE, direction fields for first-order ODEs, initial value problems
- Goal for the course: develop a basic library of ODEs we can solve exactly, and hint at a strategy for ODEs we can't solve.

2 First order ODEs

- Separable ODEs
- Integrating factors for linear, first-order ODEs
- Modeling (lots of salt tanks)
- Structure of the solution set for a linear ODE, and for homogeneous linear ODEs.
- Existence and uniqueness for solutions to IVPs
- Autonomous first-order ODEs: phase portraits, stability of solutions, and applications to population dynamics.

3 Systems of first order linear ODEs with constant coefficients

- Fundamental solution sets for homogeneous systems
- Constructing fundamental solution sets via eigensystems:
  - distinct real eigenvalues
  - distinct complex eigenvalues
  - repeated eigenvalues
- Phase portraits for 2D systems
- Shifted linear systems (i.e., $\mathbf{x}' = A(\mathbf{x} - \mathbf{a})$)
- Applications (e.g. systems of salt tanks)

4 Second order ODEs

- Converting a second order ODE to a 2D system of first order ODEs.
- Solving second order linear homogeneous ODEs with constant coefficients. Three cases:
  - distinct real eigenvalues
  - distinct complex eigenvalues

  - repeated eigenvalues
- Free vibrations (i.e., spring-mass systems with no external forcing)
- Non-homogeneous second order linear ODEs with constant coefficients via the method of undetermined coefficients.
- Non-homogeneous second order linear ODEs via variation of parameters. (Here the coefficients might not be constant.)
- Forced vibrations (i.e., spring-mass systems with external forcing)
- Resonance, frequency-response function, gain.

5 Using the Laplace transform to solve IVPs

- Outline of the strategy: $\mathcal{L}$ turns IVPs into algebra problems, and $\mathcal{L}^{-1}$ turns algebraic solutions into IVP solutions.
- Computing the Laplace transform from the definition.
- Various formulae for $\mathcal{L}$: how it treats derivatives, time-shifts, multiplication by e^{ct}, etc.
- Skills for computing $\mathcal{L}^{-1}$ — especially partial fractions.
- Strange forcing terms: discontinuous functions, periodic functions, impulse functions.
- Convolution and impulse response. Use these to break an IVP solution into a forced response and a free response.

6 Linearization

- Autonomous systems and critical points.
- Linearizing autonomous systems at their critical points.
- When does the stability of the linear system's CP match the stability as a CP of the original system?
- For 2D linear systems with constant coefficients, determining stability based on trace and determinant.
- Applications: competing species systems and predator-prey systems.

7 Numerical methods

- Euler's method for approximating solutions to first-order ODEs.
- Accuracy of Euler's method: computing bounds for local truncation error and global truncation error.
- Improved Euler method.
- Runge-Kutta method.
- Applying any of these methods to first-order systems, or to higher-order ODEs.